

PS 376 : Analysis of International Relations

Section Handout - Week 6

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121 North Hall

mywebspace.wisc.edu/ohls/web/376.html

Tues 4-5, Weds 11-12, or by appointment

1 Classic 2x2 Games

Reference Here are a few of the classic 2x2 normal form games you may come across:

<p><u>PD</u> 2</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td style="text-align: center;">C</td><td style="text-align: center;">D</td></tr> <tr><td style="text-align: center;">1</td><td style="border: 1px solid black; padding: 2px;">3,3</td><td style="border: 1px solid black; padding: 2px;">1,4</td></tr> <tr><td style="text-align: center;">D</td><td style="border: 1px solid black; padding: 2px;">4,1</td><td style="border: 1px solid black; padding: 2px;">2,2</td></tr> </table>		C	D	1	3,3	1,4	D	4,1	2,2	<p><u>Chicken</u> 2</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td style="text-align: center;">Sw</td><td style="text-align: center;">Str</td></tr> <tr><td style="text-align: center;">1</td><td style="border: 1px solid black; padding: 2px;">3,3</td><td style="border: 1px solid black; padding: 2px;">2,4</td></tr> <tr><td style="text-align: center;">Str</td><td style="border: 1px solid black; padding: 2px;">4,2</td><td style="border: 1px solid black; padding: 2px;">1,1</td></tr> </table>		Sw	Str	1	3,3	2,4	Str	4,2	1,1	<p><u>Harmony</u> 2</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td style="text-align: center;">C</td><td style="text-align: center;">D</td></tr> <tr><td style="text-align: center;">1</td><td style="border: 1px solid black; padding: 2px;">4,4</td><td style="border: 1px solid black; padding: 2px;">2,3</td></tr> <tr><td style="text-align: center;">D</td><td style="border: 1px solid black; padding: 2px;">3,2</td><td style="border: 1px solid black; padding: 2px;">1,1</td></tr> </table>		C	D	1	4,4	2,3	D	3,2	1,1	<p><u>Deadlock</u> 2</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td style="text-align: center;">C</td><td style="text-align: center;">D</td></tr> <tr><td style="text-align: center;">1</td><td style="border: 1px solid black; padding: 2px;">2,2</td><td style="border: 1px solid black; padding: 2px;">1,4</td></tr> <tr><td style="text-align: center;">D</td><td style="border: 1px solid black; padding: 2px;">4,1</td><td style="border: 1px solid black; padding: 2px;">3,3</td></tr> </table>		C	D	1	2,2	1,4	D	4,1	3,3
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2 Mixed Strategies

Definition: Mixed Strategy A strategy in which the player places some positive probability on more than one action (i.e. does not play one action for certain, but rather sometimes does one and sometimes does another).

- Note: On any given move, the player will choose *randomly* from among the strategies specified, according to the probability distribution specified. If the probability is $\frac{1}{2}$ imagine flipping a coin, if the probability is $\frac{1}{6}$ imagine rolling a die, etc. If the player is simply alternating, or taking different moves in predictable turns, it is not a mix, and the strategic dynamics are different.
- Note: Technically, all strategies are mixed strategies. A pure strategy (also known as a *degenerate* mixed strategy) is simply a mix that places probability 1 on a certain action and probability 0 on all others. For purposes of this class, we will use the term ‘mixed strategy’ to refer solely to mixes that include more than one action played with positive probability (also known as *non-degenerate* mixed strategies).

Definition: Mixed Strategy Nash Equilibrium Any Nash equilibrium in which one or more players are playing mixed strategies.

- Note: In order for playing a mixed strategy to be a best response, the player must be indifferent between all options included in his mix (given what the other player is doing). If, holding other strategies constant, he strictly prefers one option to another, then he would never play the less-preferred strategy - there would be a profitable deviation to playing solely the more-preferred strategy. The complementary relationship is also true: the player’s mix must leave the opponent indifferent between her strategies (assuming she is also mixing).
- Note: The probability distribution a player will play is determined by the *other* player’s payoffs, since each must mix at just the right level to leave the other indifferent (and since each, being indifferent, does not care what mix they play for purposes of their own payoffs).
- Note: Because we are now dealing with the expected value of probability distributions over different possible outcomes, the cardinal value of payoffs now carries meaningful information. Receiving utility 100 for outcome A and 1 for outcome B is now different than receiving utility 4 for outcome A and 3 for outcome B.

Example To solve for mixed strategy Nash equilibria: First, check for strictly dominated strategies. If they exist, they can be ruled out - strictly dominated strategies will not be played with positive probability in any pure or mixed strategy Nash equilibrium. Then, assign probability p to the top strategy for player 1 (and probability $1-p$ to the bottom strategy), and determine what value of p would make player 2 indifferent between playing her left and right strategies. Finally, assign probability q to the left strategy for player 2 (and probability $1-q$ to the right strategy), and determine what value of q would make player 1 indifferent between playing his top and bottom strategies.

		2	
		C (q)	D ($1-q$)
1	A (p)	2,2	0,0
	B ($1-p$)	0,0	1,1

$$\begin{aligned}
 u_2(C) &= u_2(D) \\
 2(p) + 0(1-p) &= 0(p) + 1(1-p) \\
 2p &= 1-p \\
 3p &= 1 \\
 p &= \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 u_1(A) &= u_1(B) \\
 2(q) + 0(1-q) &= 0(q) + 1(1-q) \\
 2q &= 1-q \\
 3q &= 1 \\
 q &= \frac{1}{3}
 \end{aligned}$$

Player 1 must be playing A with probability $\frac{1}{3}$ and B with probability $\frac{2}{3}$ to leave player 2 indifferent.

Player 2 must be playing C with probability $\frac{1}{3}$ and D with probability $\frac{2}{3}$ to leave player 1 indifferent.

Thus, the unique mixed strategy Nash equilibrium is $((\frac{1}{3}A + \frac{2}{3}B), (\frac{1}{3}C + \frac{2}{3}D))$. There are also two pure strategy Nash equilibria: (A,C) and (B,D).

Exercise 1 Find all mixed strategy equilibria in the games below.

		2	
		L	R
1	U	2,7	3,5
	D	1,1	4,2

(a)

		2	
		I	O
1	T	4,0	0,1
	B	3,9	5,4

(b)

Exercise 2 Now lets make things a bit more interesting. Find all mixed strategy equilibria in the games below.

		Defender	
		Land	Air
Attacker	Land	-2,2	1,-4
	Air	4,-3	2,-1

(c)

		Govt	
		Bailout	Let Fail
Banks	Risky	6,2	0,3
	Safe	3,6	4,5

(d)

		South Korea	
		Trade	Sanctions
North Korea	No Nuclear	3,7	1,4
	Energy	5,5	4,3
	Weapons	7,1	2,4

(e)